

Name: _____

Date: March 3, 2026

Lunar Eclipse

Most of the US witnessed the lunar eclipse in the early hours today (Tuesday March 3, 2026). Unfortunately, due to the presence of the cloud Iowa City couldn't enjoy it. The next eclipse won't occur until 2029. Let us take this moment to apply our calculus knowledge to model the eclipse and gain some insights.

Lunar eclipse occurs when the earth is in between the sun and the moon and thereby casting a shadow on the latter. There are two levels of this shadow, penumbra and umbra. As the moon travels through the umbra, we get a totality where the moon appears red. The radius of the umbra is roughly 2.6 times the radius of the moon.

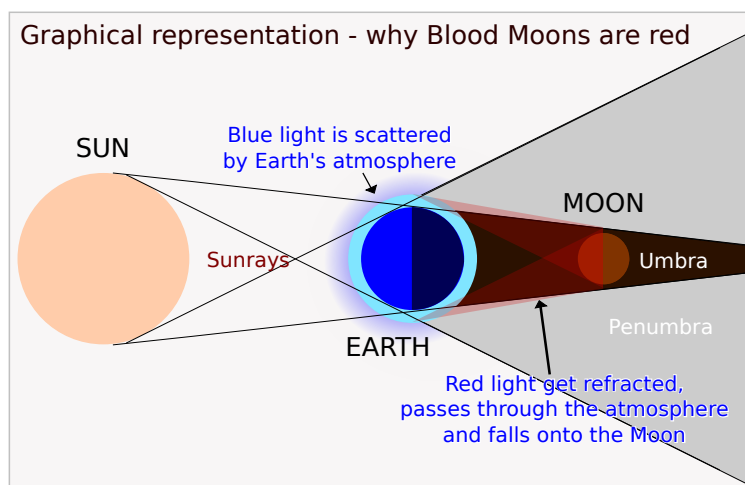


Figure 1: Blood moon.

Assumptions: We now make a simplified model so that it is easy to compute, yet retaining the essence of the actual theory.

- We assign the radius of the moon to be $r = 50$ units and that of the umbra $R = 130$ units. Note that their ratio is exactly 2.6, just like in real life.
- The moon follows an elliptic path as it travels through the shadow. We approximate it with a straight line. This is a reasonable assumption.

$$x(t) = -88t + 212 \quad \text{and} \quad y(t) = 66t - 84$$

where $(x(t), y(t))$ represents the coordinates of the center of the moon at time t . The time t is measured in hours and assume $t = 0$ represents 3 : 30 am.

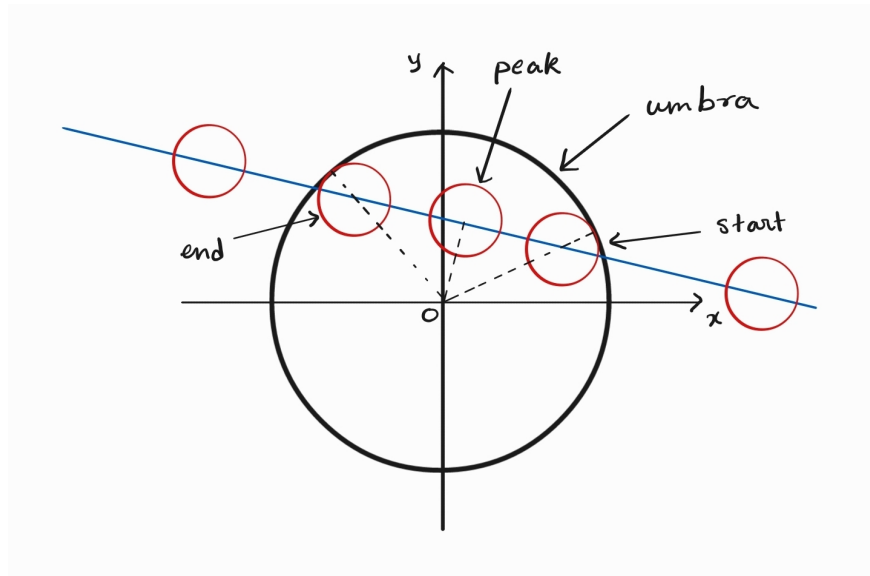


Figure 2: Moon transiting through the shadow

Task 1: Find the standard equation of the line described by the parametric equation

$$x(t) = -88t + 212, \quad y(t) = 66t - 84$$

Task 2: Assuming the shadow of the earth is centered at the origin, find the distance $D(t)$ as a function of time, from the center of the shadow to the moon.

The peak totality occurs when the moon is the closest to the center of the shadow. In our model, it is equivalent to when $D(t)$ is minimum. The point of maxima or minima of a function can be found by solving $D'(t) = 0$.

Task 3: Find the time of the peak totality.

(a) First find $D'(t)$. Chain Rule should come in handy.

(b) Solve for $D'(t) = 0$.

Let us now find the duration of the eclipse. It starts when the moon is completely inside the umbra, and ends when all of it is just outside. The distance to the center in these moments is $R - r = 130 - 50 = 80$. So we solve for $D(t) = 80$.

Task 4: Solve $D(t) = 80$. Note $D(t)$ is a quadratic polynomial, so it has at most two solutions. In this case we expect the two solutions to be distinct, indicating the start and the end of the totality.

Task 5: Conclude by summarizing your findings. Write a short paragraph (4-5 sentences) to be reported in a local daily for the public's awareness of the eclipse's timings.